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Adham

Fall 2010-2011	**Final Exam**	**Date: February 3**
Prof: M. Egeileh	**Math 201 - Sections 24 to 26**	**Duration: 2 hours**

Problem 1 *(answer on pages* **1** *and* **2** *of the booklet)*
Which of the following series converge, and which diverge? *(6 pts each)*

a) $\sum_{n=1}^{\infty} \frac{(\ln n)^{201}}{n^{1.02}}$ b) $\sum_{n=1}^{\infty} (-1)^{n-1}\left(1+\frac{8}{n}\right)^{n}$ c) $\sum_{n=1}^{\infty} \sqrt{n}\ln\left(1+\frac{1}{n^{1.5}}\right)$

$n=0$

Problem 2 *(answer on page* **3** *of the booklet)* Show that the vector field

$$\vec{F} = (2x\cos y + yz)\,\vec{i} + (xz - x^2\sin y)\,\vec{j} + (xy)\,\vec{k}$$

is conservative, and evaluate the work done by $\vec{F}$ along a curve joining $(5,0,9)$ to $(1,\pi,0)$. *(24 pts)*

Problem 3 *(answer on pages* **4** *and* **5** *of the booklet)*
Consider the function $f(x,y) = x^2 + 2y^2 - \frac{y^3}{3}$.

1. Find all local maxima, local minima, and saddle points of $f(x,y)$. *(13 pts)*
2. Find the tangent plane and normal line to the surface $z = f(x,y)$ at the point $(0, 4, \frac{32}{3})$. *(12 pts)*

Problem 4 *(answer on page* **6** *of the booklet)* Suppose $f(x,y,z)$ is a differentiable function of two variables such that : $\vec{\nabla} f(3,2,1) = 6\vec{i} - 2\vec{j}$, $\vec{\nabla} f(3,1,-4) = \vec{i} + \vec{j} + \vec{k}$ and $\vec{\nabla} f(2,1,7) = 3\vec{i} - \vec{j} + \vec{k}$. Let $x = 2r + s$, $y = 2r - s$, $z = -2(r^2 + s^2)$ and $w = f(x,y,z)$. Find w_r and w_s at the point $(r,s) = (1,1)$. *(24 pts)*

Problem 5 *(answer on pages* **7** *and* **8** *of the booklet)* Let D be the region bounded from below by the cone $z = \sqrt{x^2+y^2}$, and from above by the paraboloid $z = 2 - x^2 - y^2$.

1. Set up and evaluate the iterated integral in cylindrical coordinates that gives the volume of D using the order of integration $dz\,dr\,d\theta$. *(11 pts)*
2. Set up (without evaluating) the iterated integral in cylindrical coordinates that gives the volume of D using the order of integration $dr\,dz\,d\theta$. *(9 pts)*
3. Set up (without evaluating) the iterated integral in spherical coordinates that gives the volume of D using the order of integration $d\rho\,d\phi\,d\theta$. *(6 pts)*

Problem 6 *(answer on pages* **9** *and* **10** *of the booklet)* Use the transformation $u = x+y$, $v = y-2x$ to rewrite $\int_0^1\int_0^{1-x} \sqrt{x+y}\,(y-2x)^2\,dy\,dx$ as an integral over an appropriate region G in the uv-plane. Then evaluate the uv-integral over G. *(25 pts)*

Problem 7 *(answer on pages* **11, 12, 13** *and* **14** *of the booklet)*
We consider the vector field $\vec{F} = \frac{x}{x^2+y^2}\vec{i} + \frac{y}{x^2+y^2}\vec{j}$. Let C_1 be the circle of center $(0,0)$ and radius 1, oriented counterclockwise. Let C_2 be the square of vertices $A(-2,2)$, $B(2,2)$, $C(2,-2)$ and $D(-2,-2)$, oriented clockwise. Finally, let R be the region of the plane inside C_2 and outside C_1.

1. Calculate the flux integral $\oint_{C_1} \vec{F}\cdot\vec{n}\,ds$ directly, by choosing a suitable parametrization for C_1. *(12 pts)*

2. Calculate the flux integral $\oint_{C_2} \vec{F} \cdot \vec{n}\, ds$ directly, by choosing a suitable parametrization for each of the four sides of C_2 ($[AB]$, $[BC]$, $[CD]$ and $[DA]$). *(8 pts)*

3. If $\vec{G}$ is a vector field in the plane, we define the **flux of $\vec{G}$ inwards the region** R by

$$\Phi(\vec{G}) = \oint_{C_1} \vec{G} \cdot \vec{n}\, ds + \oint_{C_2} \vec{G} \cdot \vec{n}\, ds$$

Use Green's theorem to show that for any vector field $\vec{G}$ in the plane,

$$\Phi(\vec{G}) = -\iint_R \operatorname{div}(\vec{G})\, dA(x,y)$$

(Hint : call D_2 the square region enclosed by C_2, and call D_1 the disk enclosed by C_1...) (6 pts)

4. Use the results of questions 1. and 3. to recalculate $\oint_{C_2} \vec{F} \cdot \vec{n}\, ds$. *(8 pts)*

Problem 8 *(answer on pages* **15, 16** *and* **last** *of the booklet)*

1. Prove that

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

for all $x \in \mathbb{R}$. *(6 pts)*

2. Approximate $\int_0^{0.1} e^{-x^2}\, dx$ with an error of magnitude less than 10^{-5}. *(6 pts)*

3. Show that

$$\int_0^{\infty} e^{-\pi x^2}\, dx = \frac{1}{2}$$

(Hint : if $I = \int_0^{\infty} e^{-\pi x^2}\, dx$, then $I^2 = \int_0^{\infty}\int_0^{\infty} e^{-\pi(x^2+y^2)}\, dx\, dy$). (6 pts)

4. Let E be the error resulting from the approximation $\int_0^{100} e^{-\pi x^2}\, dx \simeq \frac{1}{2}$. Show that

$$|E| < \frac{e^{-5000\pi}}{2}$$

(6 pts)